

Funções de Uma Variável II - Lista de Exercícios nº 3
Aula 4

Observação. Os exercícios indicados foram retirados do livro Calculus, volume 1, 7ª edição, editora Cengage Learning, de J. Stewart. Encontram-se no site stewartcalculus.com dicas (“homework hints”) para resolução dos exercícios numerados em vermelho.

Seção 7.4: Integração de Funções Racionais por Frações Parciais.

EXAMPLE 7 Write out the form of the partial fraction decomposition of the function

$$\frac{x^3 + x^2 + 1}{x(x-1)(x^2+x+1)(x^2+1)^3}$$

SOLUTION

$$\begin{aligned} & \frac{x^3 + x^2 + 1}{x(x-1)(x^2+x+1)(x^2+1)^3} \\ &= \frac{A}{x} + \frac{B}{x-1} + \frac{Cx+D}{x^2+x+1} + \frac{Ex+F}{x^2+1} + \frac{Gx+H}{(x^2+1)^2} + \frac{Ix+J}{(x^2+1)^3} \end{aligned}$$

1–6 Write out the form of the partial fraction decomposition of the function (as in Example 7). Do not determine the numerical values of the coefficients.

1. (a) $\frac{1+6x}{(4x-3)(2x+5)}$

(b) $\frac{10}{5x^2-2x^3}$

2. (a) $\frac{x}{x^2+x-2}$

(b) $\frac{x^2}{x^2+x+2}$

3. (a) $\frac{x^4+1}{x^5+4x^3}$

(b) $\frac{1}{(x^2-9)^2}$

4. (a) $\frac{x^4-2x^3+x^2+2x-1}{x^2-2x+1}$

(b) $\frac{x^2-1}{x^3+x^2+x}$

5. (a) $\frac{x^6}{x^2-4}$

(b) $\frac{x^4}{(x^2-x+1)(x^2+2)^2}$

6. (a) $\frac{t^6+1}{t^6+t^3}$

(b) $\frac{x^5+1}{(x^2-x)(x^4+2x^2+1)}$

7–38 Evaluate the integral.

7. $\int \frac{x^4}{x-1} dx$

8. $\int \frac{3t-2}{t+1} dt$

9. $\int \frac{5x+1}{(2x+1)(x-1)} dx$

10. $\int \frac{y}{(y+4)(2y-1)} dy$

11. $\int_0^1 \frac{2}{2x^2+3x+1} dx$

12. $\int_0^1 \frac{x-4}{x^2-5x+6} dx$

13. $\int \frac{ax}{x^2-bx} dx$

14. $\int \frac{1}{(x+a)(x+b)} dx$

15. $\int_3^4 \frac{x^3-2x^2-4}{x^3-2x^2} dx$

16. $\int_0^1 \frac{x^3-4x-10}{x^2-x-6} dx$

17. $\int_1^2 \frac{4y^2-7y-12}{y(y+2)(y-3)} dy$

18. $\int \frac{x^2+2x-1}{x^3-x} dx$

19. $\int \frac{x^2+1}{(x-3)(x-2)^2} dx$

20. $\int \frac{x^2-5x+16}{(2x+1)(x-2)^2} dx$

21. $\int \frac{x^3+4}{x^2+4} dx$

22. $\int \frac{ds}{s^2(s-1)^2}$

23. $\int \frac{10}{(x-1)(x^2+9)} dx$

24. $\int \frac{x^2-x+6}{x^3+3x} dx$

25. $\int \frac{4x}{x^3+x^2+x+1} dx$

26. $\int \frac{x^2+x+1}{(x^2+1)^2} dx$

27. $\int \frac{x^3+x^2+2x+1}{(x^2+1)(x^2+2)} dx$

28. $\int \frac{x^2-2x-1}{(x-1)^2(x^2+1)} dx$

29. $\int \frac{x+4}{x^2+2x+5} dx$

30. $\int \frac{3x^2+x+4}{x^4+3x^2+2} dx$

31. $\int \frac{1}{x^3-1} dx$

32. $\int_0^1 \frac{x}{x^2+4x+13} dx$

33. $\int_0^1 \frac{x^3+2x}{x^4+4x^2+3} dx$

34. $\int \frac{x^3+x-1}{x^3+1} dx$

35. $\int \frac{dx}{x(x^2+4)^2}$

36. $\int \frac{x^4+3x^2+1}{x^5+5x^3+5x} dx$

37. $\int \frac{x^2-3x+7}{(x^2-4x+6)^2} dx$

38. $\int \frac{x^3+2x^2+3x-2}{(x^2+2x+2)^2} dx$

39–52 Make a substitution to express the integrand as a rational function and then evaluate the integral.

39. $\int \frac{\sqrt{x+1}}{x} dx$

40. $\int \frac{dx}{2\sqrt{x+3} + x}$

41. $\int \frac{dx}{x^2 + x\sqrt{x}}$

42. $\int_0^1 \frac{1}{1 + \sqrt[3]{x}} dx$

43. $\int \frac{x^3}{\sqrt[3]{x^2+1}} dx$

44. $\int_{1/3}^3 \frac{\sqrt{x}}{x^2+x} dx$

45. $\int \frac{1}{\sqrt{x} - \sqrt[3]{x}} dx$ [Hint: Substitute $u = \sqrt[6]{x}$.]

46. $\int \frac{\sqrt{1+\sqrt{x}}}{x} dx$

47. $\int \frac{e^{2x}}{e^{2x} + 3e^x + 2} dx$

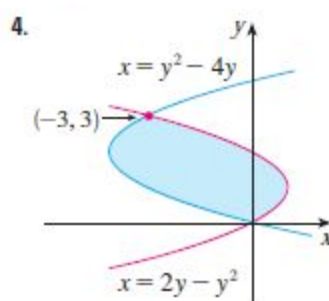
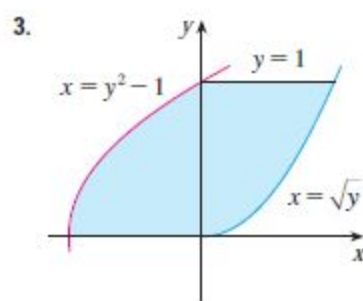
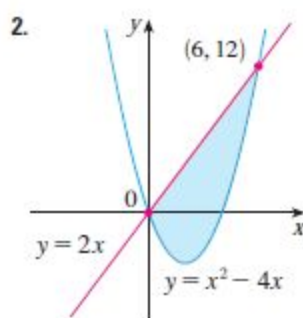
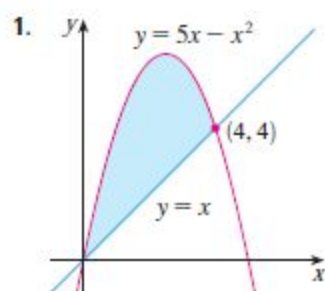
48. $\int \frac{\sin x}{\cos^2 x - 3 \cos x} dx$

49. $\int \frac{\sec^2 t}{\tan^2 t + 3 \tan t + 2} dt$

50. $\int \frac{e^x}{(e^x - 2)(e^{2x} + 1)} dx$

Seção 5.1: Áreas entre Curvas.

1–4 Find the area of the shaded region.



5–12 Sketch the region enclosed by the given curves. Decide whether to integrate with respect to x or y . Draw a typical approximating rectangle and label its height and width. Then find the area of the region.

5. $y = x + 1$, $y = 9 - x^2$, $x = -1$, $x = 2$

6. $y = \sin x$, $y = x$, $x = \pi/2$, $x = \pi$

7. $y = (x - 2)^2$, $y = x$

8. $y = x^2 - 2x$, $y = x + 4$

9. $y = \sqrt{x + 3}$, $y = (x + 3)/2$

10. $y = \sin x$, $y = 2x/\pi$, $x \geq 0$

11. $x = 1 - y^2$, $x = y^2 - 1$

12. $4x + y^2 = 12$, $x = y$

13–28 Sketch the region enclosed by the given curves and find its area.

13. $y = 12 - x^2$, $y = x^2 - 6$

14. $y = x^2$, $y = 4x - x^2$

15. $y = \sec^2 x$, $y = 8 \cos x$, $-\pi/3 \leq x \leq \pi/3$

16. $y = \cos x$, $y = 2 - \cos x$, $0 \leq x \leq 2\pi$

17. $x = 2y^2$, $x = 4 + y^2$

18. $y = \sqrt{x - 1}$, $x - y = 1$

19. $y = \cos \pi x$, $y = 4x^2 - 1$

20. $x = y^4$, $y = \sqrt{2 - x}$, $y = 0$

21. $y = \cos x$, $y = 1 - 2x/\pi$

22. $y = x^3$, $y = x$

23. $y = \cos x$, $y = \sin 2x$, $x = 0$, $x = \pi/2$

24. $y = \cos x$, $y = 1 - \cos x$, $0 \leq x \leq \pi$

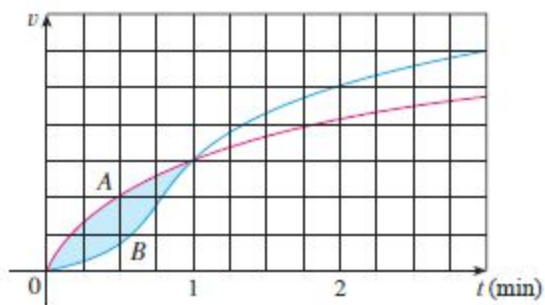
25. $y = \sqrt{x}$, $y = \frac{1}{2}x$, $x = 9$

26. $y = |x|$, $y = x^2 - 2$

27. $y = 1/x^2$, $y = x$, $y = \frac{1}{8}x$

28. $y = \frac{1}{4}x^2$, $y = 2x^2$, $x + y = 3$, $x \geq 0$

47. Two cars, A and B, start side by side and accelerate from rest. The figure shows the graphs of their velocity functions.
- Which car is ahead after one minute? Explain.
 - What is the meaning of the area of the shaded region?
 - Which car is ahead after two minutes? Explain.
 - Estimate the time at which the cars are again side by side.



50. Find the area of the region bounded by the parabola $y = x^2$, the tangent line to this parabola at $(1, 1)$, and the x -axis.